Lesson 11A: Solution Sets For Equations and Inequalities

Classwork

Exercise 1

* 1. Consider the statement: “The President of the United States is a United States citizen.”

Is the statement a grammatically correct sentence?

What is the subject of the sentence?

What is the verb in the sentence?

And what is the object of the sentence?

And finally, is the sentence true?

* 1. Consider the statement: “The President of France is a United States citizen.”

Is the statement a grammatically correct sentence?

What is the subject of the sentence?

What is the verb in the sentence?

And what is the object of the sentence?

And finally, is the sentence true?

* 1. Consider the statement: “2 + 3 = 1 + 4.”

This is a sentence. What is the verb of the sentence?

What is the subject of the sentence and what is the object?

Is the sentence true?

* 1. Consider the statement: “2 + 3 = 9 + 4."

Is this statement a sentence? And if so, is the sentence true or false?

A *number sentence* is a statement of equality between two numerical expressions.

A number sentence is said to be *true* if both numerical expressions are equivalent (that is, both evaluate to the same number). It is said to be *false* otherwise. True and false are called *truth values*.

Exercise 2

Determine whether the following number sentences are TRUE or FALSE.

d.

j.

Exercise 3

* 1. Could a number sentence be both TRUE and FALSE?
  2. Could a number sentence be neither TRUE nor FALSE?

An ***algebraic equation*** is a statement of equality between two expressions.

Algebraic equations can be number sentences (when both expressions are numerical), but often they contain symbols whose values have not been determined.

**Example 1**

Consider the equation, , where represents a real number.

Are the expressions and algebraically equivalent?

* 1. The following table shows how we might “sift” through various values to assign to the variable symbol in the hunt for values that would make the equation true.

|  |  |  |
| --- | --- | --- |
| -VALUE | THE EQUATION | TRUTH VALUE |
| Let |  |  |
| Let |  |  |
| Let |  |  |
| Let |  |  |
| Let |  |  |
| Let |  |  |
| Let |  |  |
| Let |  |  |

b. Is there another solution?

The ***solution set***of an equation written with only one variable is the set of all values one can assign to that variable to make the equation a true statement. Any one of those values is said to be a ***solution to the equation****.*

To ***solve an equation*** means to ***find the solution set*** for that equation.

c. Present the solution set in words, in set notation and also graphically

WORDS:

SET NOTATION:

GRAPHICALLY:



**Example 2**

One can describe a solution set in any of the following ways:

**IN WORDS:**  has solutions and . (That is, is true when or .)

**IN SET NOTATION:**  The solution set of is .

**IN A GRAPHICAL REPRESENTATION ON A NUMBER LINE:** The solution set of is:



In this graphical representation, a solid dot is used to indicate a point on the number line that is to be included in the solution set. (WARNING: The dot one physically draws is larger than the point it represents. One hopes that it is clear from the context of the diagram which point each dot refers to.)

How set notation works.

* The curly brackets { } indicate we are denoting a set. A set is essentially a collection of things, e.g., letters, numbers, cars, people. In this case, the things are numbers.
* From this example, the numbers and are called elements of the set. No other elements belong in this particular set because no other numbers make the equation true.
* When elements are listed, they are listed in increasing order.
* Sometimes, a set is empty; it has no elements. In which case, the set looks like { }. We often denote this with the symbol, . We refer to this *as* *the empty set* or *the null set*.

Solve for : .

Exercise 1

Solve for : . Present the solution set in words, in set notation and also graphically.

Exercise 2

Depict the solution set of in words, in set notation, and graphically.

WORDS:

SET NOTATION:

GRAPHICALLY:

**Example 3**

Solve for , over the set of positive real numbers.

|  |  |  |
| --- | --- | --- |
| -VALUE | THE EQUATION | TRUTH VALUE |
| Let |  | TRUE |
| Let |  | TRUE |
| Let |  | TRUE |
| Let |  | TRUE |
| Let |  | TRUE |
| Let |  | TRUE |

Is there a time when the value of x is false?

**Exercise 3**

Solve for , over the set of all non-zero real numbers. Describe the solution set in words, in set notation, and graphically.

WORDS:

SET NOTATION:

GRAPHICALLY:

Problem Set 11A

1. Which of the following are algebraic equations?

2. Determine whether the following number sentences are true or false.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

3. In the following equations, let . Determine whether the following equations are true, false, or neither true nor false.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |

Lesson 11B: Solution Sets For Equations and Inequalities

**Example 4**

Solve for :

Describe the solution set in words, in set notation, and graphically.

WORDS:

SET NOTATION:

GRAPHICALLY:

An **identity** is an equation that is always true.

Exercise 5

Identify the properties of arithmetic that justify why each of the following equations has a solution set of all real numbers:

Exercise 6

Create an expression for the right side of each equation such that the solution set for the equation will be all real numbers. (There is more than one possibility for each expression. Feel free to write several answers for each one.)

* 1. =
  2. =
  4. =

Example 5

Solve for ,

Describe the solution set in words, in set notation, and graphically.

WORDS:

SET NOTATION:

GRAPHICALLY:

Exercise 7

Solve for : . Describe the solution set in words, in set notation, and graphically.

WORDS:

SET NOTATION:

GRAPHICALLY:

Lesson Summary

The ***solution set***of an equation written with only one variable symbol is the set of all values one can assign to that variable to make the equation a true number sentence. Any one of those values is said to be a ***solution to the equation****.*

To ***solve an equation*** means to ***find the solution set*** for that equation.

One can describe a solution set in any of the following ways:

**IN WORDS**: has solutions and . (That is, is true when or .)

**IN SET NOTATION**: The solution set of is .

It is awkward to express the set of infinitely many numbers in set notation. In these cases we can use the   
notation: . For example reads, “ is a real number where is greater than zero. The symbol can be used to indicate all real numbers.

**IN A GRAPHICAL REPRESENTAION ON A NUMBER LINE**: The solution set of is:



In this graphical representation, a solid dot is used to indicate a point on the number line that is to be included in the solution set. (WARNING: The dot one physically draws is larger than the point it represents! One hopes that it   
is clear from the context of the diagram which point each dot refers to.)

Problem Set 11B

For problems 1–4, the two expressions are algebraically equivalent. State the property (or properties) that justifies the equivalence.

2.

3.

4. (xy)z = (zx)y

Fill in the chart below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | **SOLUTION SET IN WORDS** | **SOLUTION SET IN SET NOTATION** | **GRAPH** |
| 5. |  |  |  |  |
| 6. |  |  |  |  |
| 7. |  |  |  |  |
| 8. |  |  |  |  |
| 9. |  |  |  |  |
| 10. |  |  |  |  |
| 11. |  |  |  |  |
| 12. |  |  |  |  |
| 13. |  |  |  |  |
| 14. |  |  |  |  |

For each solution set graphed below, (a) describe the solution set in words, (b) describe the solution set in set notation, and (c) write an equation or an inequality that has the given solution set.

|  |  |  |  |
| --- | --- | --- | --- |
| 15. | http://img.sparknotes.com/figures/5/50ca5e784bb7e4242910d5b8a571d103/number_line.gif | 16. | http://img.sparknotes.com/figures/5/50ca5e784bb7e4242910d5b8a571d103/number_line.gif |
|  | (a) |  | (a) |
|  | (b) |  | (b) |
|  | (c) |  | (c) |
| 17. | http://img.sparknotes.com/figures/5/50ca5e784bb7e4242910d5b8a571d103/number_line.gif | 18. | **http://img.sparknotes.com/figures/5/50ca5e784bb7e4242910d5b8a571d103/number_line.gif** |
|  | (a) |  | (a) |
|  | (b) |  | (b) |
|  | (c) |  | (c) |
| 19. | **http://img.sparknotes.com/figures/5/50ca5e784bb7e4242910d5b8a571d103/number_line.gif** | 20. | **http://img.sparknotes.com/figures/5/50ca5e784bb7e4242910d5b8a571d103/number_line.gif** |
|  | (a) |  | (a) |
|  | (b) |  | (b) |
|  | (c) |  | (c) |
| 21. | **http://img.sparknotes.com/figures/5/50ca5e784bb7e4242910d5b8a571d103/number_line.gif** | 22. | **http://img.sparknotes.com/figures/5/50ca5e784bb7e4242910d5b8a571d103/number_line.gif** |
|  | (a) |  | (a) |
|  | (b) |  | (b) |
|  | (c) |  | (c) |